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# Calculating the Position and Velocity Components of HST

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**Abstract.** The *HST* uses an onboard model of its orbit to perform in real-time various control functions relating to spacecraft operations and observation support. The flight software uses the equation of center to solve Kepler's equation for two bodies. New coefficients for the ephemeris are updated every other day, and are archived as FITS keywords for each *HST* observation data set. Using these coefficients,an observer can perform a variety of calibration and characterization calculations relating to the orbital position and velocity of the telescope. We present here the methodology for computing the *HST* state vectors using this information.

## 1. Introduction

Many precise measurements with the *Hubble Space Telescope* (*HST*) require knowledge of its position and velocity during the observations. Orbital parallax, velocity aberration, Doppler shift, and light-travel time can all be significant when converting *HST* observations to the geocentric system, and from there to the barycenter of the solar system. In addition, the quality of observations are affected by the near-earth environment in which *HST* operates. An observer would be wise to understand such effects as scattered earth light, radiation background, and geomagnetically-induced motion on the data.

One way to determine the motion of *HST* during an observation is through the definitive orbit files that are archived at the Space Telescope Science Institute (ST ScI). Every other day the Flight Dynamics Facility (FDF) at Goddard Space Flight Center computes the position and velocity of the *HST* for the previous two days based on ranging measurements of the spacecraft. This information is forwarded to the ST ScI in the form of a list of *HST* state vectors for each minute of time, with each record giving the J2000 rectangular components of the position, in km, and velocity, in km $s^{-1}$, in the geocentric inertial coordinate system. The observer must find and extract the appropriate file(s) from the archive and interpolate the *HST* state vector data to the relevant times of the observations.

An easier method is to use the onboard ephemeris parameters that are provided with the data sets themselves. When the definitive orbit file is generated by the FDF, a set of ephemeris coefficients is created. The *HST* flight software uses these coefficients for various spacecraft control functions. During pipeline processing at the ST ScI, these are archived in the `*.shh` (non-astrometry) or

`*.dbm` (astrometry) header file for each observation. We summarize using this information to compute the position and velocity of the *HST*.

Table 1. Onboard Ephemeris Model Parameters

| FITS Keyword | FITS Description | Symbol |
|---|---|---|
| EPCHTIME | epoch time of parameters (secs since 1/1/85) | $\tau$ |
| MEANANOM | mean anomaly (radians) | $\mathrm{M}_0$ |
| FDMEANAN | 1st derivative coef for mean anomly (revs/sec) | $\dot{\mathrm{M}}$ |
| SDMEANAN | 2nd deriv coef for mean anomaly (revs/sec/sec) | $\ddot{\mathrm{M}}$ |
| ECCENTRY | eccentricity | $e$ |
| SEMILREC | semi-latus rectum (meters) | $a(1-e^2)$ |
| RASCASCN | right ascension of ascending node (revolutions) | $\Omega_0$ |
| RCASCNRV | rt chge right ascension ascend node (revs/sec) | $\dot{\Omega}$ |
| ARGPERIG | argument of perigee (revolutions) | $\omega_0$ |
| RCARGPER | rate change of argument of perigee (revs/sec) | $\dot{\omega}$ |
| COSINCLI | cosine of inclination | $\cos i$ |
| SINEINCL | sine of inclination | $\sin i$ |
| CIRVELOC | circular orbit linear velocity (meters/second) | $\mathrm{V_C}$ |
| TIMEFFEC | time parameters took effect (secs since 1/1/85) | – |

## 2. Computations

The *HST* travels in a nearly circular orbit, with an altitude of about 600 km and velocity of $7.5\,\mathrm{km\,s^{-1}}$. The orbital model used onboard is based on a simple two-body system, with perturbations to certain Keplerian elements due to the proximity of the earth. In Table 1 we list the relevant FITS keywords and descriptions that can be found in the header files, as well as the symbols used in the equations below.

The steps to compute the geocentric, rectangular coordinates for the *HST* using the parameters in Table 1 are as follows. First, the observer should verify that the correct onboard ephemeris has been archived with the data. The TIMEFFEC keyword specifies the beginning time at which the parameters are valid and its value should be within 2–3 days before the start of the observations.

For a time of interest, $t$, calculate the mean anomaly, M, from the initial position, $\mathrm{M}_0$, at the epoch time of the parameters, $\tau$,

$$\mathrm{M} = \mathrm{M}_0 + 2\pi\left[\dot{\mathrm{M}}(t-\tau) + \frac{1}{2}\ddot{\mathrm{M}}(t-\tau)^2\right]. \tag{1}$$

Compute the true anomaly, $\nu$, using the equation of center to solve Kepler's equation. This is typically expressed as a series in $\sin n\mathrm{M}$ and $e$. For small $e$, terms only up to $e^3$ are needed (e.g., Smart 1965, equation V-85),

$$\nu = \mathrm{M} + (2e - \frac{e^3}{4})\sin\mathrm{M} + \frac{5}{4}e^2\sin 2\mathrm{M} + \frac{13}{12}e^3\sin 3\mathrm{M}.$$

Since trigonometric functions are computationally expensive, *HST* uses a different form of this equation involving only $\sin \mathrm{M}$ and $\cos \mathrm{M}$. Collecting like terms of $e^n$ and expanding $\sin n\mathrm{M}$, one can show

$$\nu = \mathrm{M} + \sin \mathrm{M}(2e + 3e^3 \cos^2 \mathrm{M} - \frac{4}{3}e^3 \sin^2 \mathrm{M} + \frac{5}{2}e^2 \cos \mathrm{M}). \tag{2}$$

Once the true anomaly is known, then the distance from the center of the earth, $r$, is

$$r = \frac{a(1-e^2)}{1 + e\cos\nu} \quad \text{(in meters)}. \tag{3}$$

The main perturbation on the orbital elements due to the non-spherical mass distribution of the earth is the regression of the ascending node, $\Omega$, and the progression of perigee, $\omega$, (Wertz 1978). The instantaneous values at $t$ are

$$\Omega = 2\pi \left[\Omega_0 + \dot{\Omega}(t-\tau)\right] \quad \text{and} \tag{4}$$

$$\omega = 2\pi \left[\omega_0 + \dot{\omega}(t-\tau)\right]. \tag{5}$$

The geocentric *HST* position, in J2000 rectangular coordinates in meters, is then

$$\begin{aligned} x &= r\left[\cos\Omega\cos(\omega+\nu) - \cos i \sin\Omega \sin(\omega+\nu)\right], \\ y &= r\left[\sin\Omega\cos(\omega+\nu) + \cos i \cos\Omega \sin(\omega+\nu)\right], \\ z &= r \sin i \sin(\omega+\nu). \end{aligned} \tag{6}$$

The corresponding equations for radial velocity can be derived by differentiating those for position. Starting with $x$,

$$\dot{x} = \frac{x}{r}\dot{r} - r(2\pi\dot{\omega} + \dot{\nu})\left[\cos\Omega\sin(\omega+\nu) + \cos i \sin\Omega\cos(\omega+\nu)\right] - 2\pi\dot{\Omega}y.$$

To eliminate the $\dot{r}$ and $\dot{\nu}$ terms, we use a well-known property of elliptical orbits that the velocity can be represented as the vector sum of two constant velocities (Smart 1965). These are the velocity, $\mu/h$, perpendicular to the radius vector and, $\mu e/h$, perpendicular to the semimajor axis, where $\mu = \mathrm{GM}_\oplus$ and $h = r^2\dot{\nu} = \mu a(1-e^2)$. Designating $\mathrm{V_C} = \mu/h$ as the circular velocity, we have

$$\begin{aligned} r\dot{\nu} &= \mathrm{V_C}(1 + e\cos\nu) \quad \text{and} \\ \dot{r} &= e\mathrm{V_C}\sin\nu. \end{aligned}$$

The rectangular velocities can now be determined from the onboard ephemeris parameters. Defining the auxiliary variables

$$\begin{aligned} a_0 &= \tfrac{1}{r}e\mathrm{V_C}\sin\nu \quad \text{and} \\ a_1 &= \mathrm{V_C}(1 + e\cos\nu) + 2\pi\dot{\omega}r, \end{aligned}$$

we have

$$\begin{aligned} \dot{x} &= a_0x - a_1\left[\cos\Omega\sin(\omega+\nu) + \cos i\sin\Omega\cos(\omega+\nu)\right] - 2\pi\dot{\Omega}y, \\ \dot{y} &= a_0y - a_1\left[\sin\Omega\sin(\omega+\nu) - \cos i\cos\Omega\cos(\omega+\nu)\right] - 2\pi\dot{\Omega}x, \\ \dot{z} &= a_0z + a_1\sin i\cos(\omega+\nu). \end{aligned} \tag{7}$$

Equations (1)–(7) yield the geocentric state vectors for *HST*.

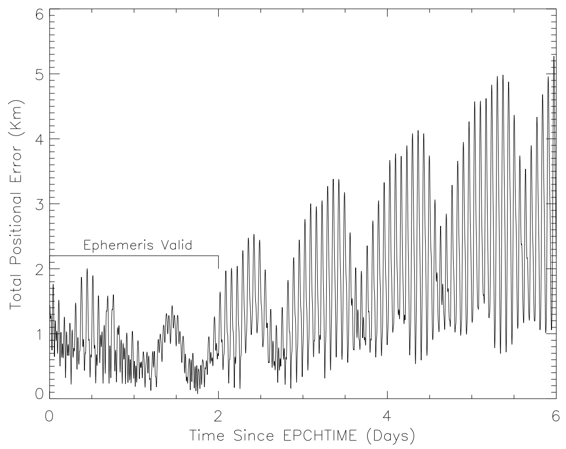


Figure 1. Typical positional errors from an on-board ephemeris. A HREF="http://hoth.stsci.edu/adass-figs/aket1.eps"¿ Original PostScript figure (82 kB) ¡/A¿¡P¿

## 3. Comparisons with Definitive Orbit Data

We can compare the results from the ephemeris model directly with the definitive orbit data. In Figure 1 we show the total error in position for several days during the first week of 1994 April. In this example the error is below 2 km for the two days during which the ephemeris is active. After this time, the error slowly increases, reaching about 5 km a week outside the two-day range of the model. The error in velocity was found to be only 0.01 km $s^{-1}$ over the whole period investigated. Comparisons for other weeks indicates that the positional error can be as high as 4 km, but the velocity error is always very small, since the orbit is nearly circular.

## 4. Conclusions

Results using the onboard ephemeris are accurate enough for most needs. A positional error of 4 km translates to an uncertainty of 1 mas at 5.5 AU from the earth, so parallax errors are small for all but nearby passing asteroids and comets. The geographic position of *HST* can be determined to better than $0\overset{\circ}{.}03$, much more accurately than needed to compute effects due to the orbital environment. The error in velocity is much smaller than can be measured with the *HST* instruments. We conclude that using the onboard ephemeris eliminates the need to import definitive orbit data. This exemplifies the value of providing users with self-documenting data sets so that further analyses can be performed without resorting to additional outside information.

## References

Smart, W. M. 1965, Spherical Astronomy (Cambridge, Cambridge University Press)

Wertz, J. R. 1978, in Spacecraft Attitude Determination and Control (Dordrecht, Reidel), p. 65