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On the Combination of Undersampled Multiframes

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Abstract. Some cameras with digital detectors, such as the Wide Field and Planetary Camera 2 onboard the *Hubble Space Telescope*, undersample the joint point spread function of telescope and camera optics. In order to overcome undersampling, two or more frames of the same field may be obtained which are shifted with respect to each other by fractions of a pixel. During data analysis such multiframes must be "registered" and "combined". This contribution investigates a novel method for merging undersampled multiframes based on projections onto convex sets (POCS). The method does not require a point-spread function; it can cope with missing data, and may incorporate prior knowledge such as non-negativity and band-limit constraints.

1. Introduction

At present the Wide Field and Planetary Camera 2 (WFPC-2) is the primary science instrument on-board *HST*. Correcting *HST*'s spherical aberration, it displays a vastly improved image quality compared to WF/PC-1. However, it's CCDs coarse sampling cannot exploit *HST*'s high spatial resolving power. Obviously science programs such as photometry of dense star fields and the morphological classification of distant galaxies would benefit from a method allowing a proper combination of two or more "dithered" WFPC-2 frames, i.e., frames that are shifted with respect to each other by fractions of a pixel.

Resolution improvement by combining (potentially undersampled) multiframes has in the recent past been investigated by a number of researchers outside astronomy (for references see Adorf 1994). Within astronomy a combination method has been proposed by Lucy (1993 and references therein) and further developed by Hook & Lucy (1993 and references therein). Being a generalization of the well-known Richardson-Lucy restoration method, this algorithm requires knowledge of the point spread function (PSF).

An alternative combination algorithm is presented here which is based on the concept of projections onto convex sets (POCS, Sezan, & Stark 1983; Youla 1987 and references therein). The proposed algorithm is similar in spirit to the POCS-method for reconstructing irregularly sampled data series (Adorf 1993). The algorithm does not presume knowledge of the PSF; it can accommodate missing data, as well as bandpass and non-negativity constraints. When a non-negativity constraint is effective, the algorithm becomes non-linear.

2. The POCS-Based Combination Algorithm

Assuming that the relative shifts between the undersampled frames are known, the POCS-algorithm (for two undersampled frames) can be stated as follows:

1. guess initial high-resolution (hi-res) estimate
2. fractionally shift hi-res frame to register it with low-resolution (lo-res) frame #1 and replace pixel values in hi-res frame by those from lo-res frame #1
3. fractionally shift hi-res frame to register it with lo-res frame #2 and replace pixel values in hi-res frame by those from lo-res frame #2
4. apply bandpass constraint
5. optionally apply non-negativity constraint
6. if not converged, iterate starting from step 2

This reconstruction algorithm belongs to the class of POCS-algorithms since each of the steps 2 to 5 is a projection onto a convex set in the linear space of all images. The iteration provably converges if there is a common point in the intersection of all convex sets. The POCS-algorithm has been implemented in the Interactive Data Language (IDL) image processing package.

3. Registration

Using fine lock on guide stars, *HST* usually achieves a pointing precision of about 7 milliarcsec RMS, or better. However, for non-contiguous observations its absolute pointing accuracy on the detector (i.e., the difference between commanded and observed pointing) has been worse. Therefore, in practice, the registration parameters have to be estimated from the data—a non-trivial problem in the presence of undersampling.

Traditionally, registration parameters have been derived from position measurements of stars contained in the field. In the context of this work, the registration parameters were estimated using a cross-correlation technique: the asymmetric shape of the correlation function in the vicinity of its peak was exploited for locating the peak to subpixel accuracy. The correlation approach has the merit of not requiring the presence of point sources in the field. While this technique has passed a few simple tests, further investigations are necessary to prove that it is a viable general method for registering "dithered" WFPC-2 frames.

A remaining problem concerns the WFPC-2 CCD-chips, which have a *non-regular* pixel grid. Single rows with narrower pixels are interlaced with the normal rows. This pattern repeats itself. Thus there is no set of registration parameters globally valid across the field of view.

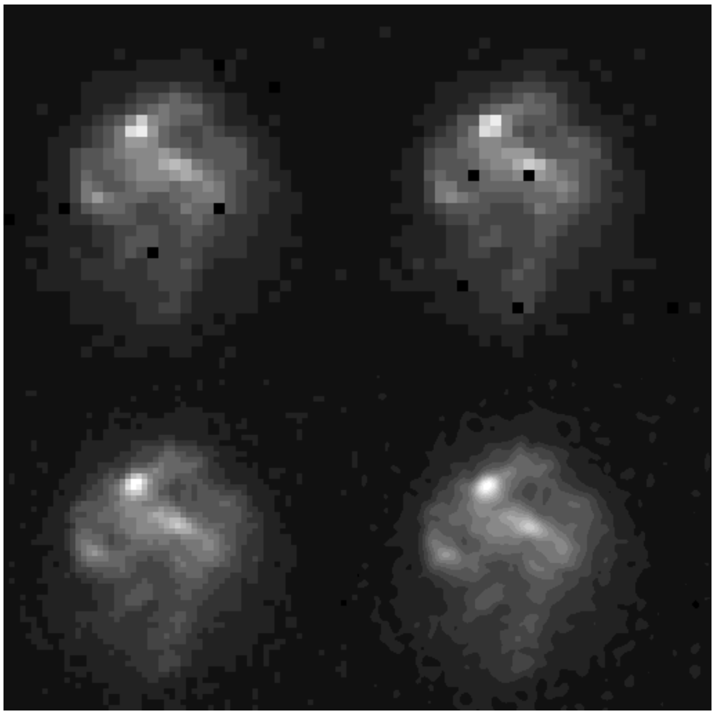

Figure 1. A "peculiar" galaxy in the distant, rich cluster CL0939+4713. **Top left:** part of a single coarsely sampled "subframe #1" at a scale of 0.1 arcsec/pixel; **top right:** part of the displaced subframe #2, shifted by $\Delta x = -0.13$ and $\Delta y = 0.31$ WFC pixels; **bottom left:** high-resolution output of the POCS-reconstruction algorithm; **bottom right:** same as before, but upsampled by a factor of four using sinc-interpolation. The combined high resolution frame reveals that the "peculiar" galaxy presumably consists of a pair of interacting galaxies.

4. Application to WFPC-2 Data

The POCS combination algorithm was applied to public *HST* Early Release Observations of the distant, rich cluster of galaxies CL0939+4713 (*HST* program number 5190). The data set has been analyzed by Dressler et al. (1994), who comment: "As remarkable as these images of systems at high redshift are, they point up the need for ... higher resolution... Substepping ... would dramatically increase the detail in these very high redshift systems ..."

The CL0939+4713 data set consists of 10 WFPC-2 exposures divided into two sets of 5 consecutive exposures. Each set was combined into a single frame using the STSDAS task "`crrej`", which simultaneously rejects cosmic ray hits. Subsequent work was carried out on chip #2 frames. The two undersampled frames in the working set still contained a large number of so-called "warm/hot pixels" at fixed CCD positions which were detected using a simple threshold algorithm. Data values at the location of warm/hot pixels were marked as "missing".

From the resulting two clean 800×800 pixel frames, two 128×128 pixel subframes centered on a peculiar galaxy were extracted (Figure 1a, b), and registered via cross-correlation. Fractional offsets of $\Delta x = -0.13$ and $\Delta y = +0.31$ WFC pixels (i.e., $\Delta x = -13$ and $\Delta y = +31$ milliarcsec, respectively) were found.

The POCS combination algorithm was applied to these subframes in a staged approach, going through a low-resolution and a medium-resolution phase to the final high resolution. Altogether 30 iterations were executed. The resulting high-resolution frame (Figure 1c, d) nicely interpolates the missing data, and reveals fine spatial structure difficult, if not impossible, to gather from the two individual undersampled frames. The contrast in the high resolution image is also increased compared to the coarsely sampled frames.

5. Summary

A novel, iterative POCS (projections onto convex sets) based algorithm has been presented for combining two or more fractionally shifted and potentially undersampled observations of the same scene into a high-resolution image. The method does not require knowledge of the point spread function, and can accommodate missing data as well as bandpass non-negativity constraints. The algorithm can also be used for solely increasing the signal-to-noise ratio without resolution enhancement. The POCS-algorithm has been successfully used to generate high resolution images from *HST* WFPC-2 observations of the distant, faint galaxy cluster CL0939+4713, revealing morphological detail which cannot be easily discerned in the individual coarsely sampled observations.

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