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An Approach for Obtaining Polarization Information from COBE-DMR

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Abstract. NASA's *Cosmic Background Explorer* (*COBE*)[4] Differential Microwave Radiometers (DMR) mapped the anisotropy of the Cosmic Microwave Background. The instrument is sensitive to linear polarization. The extraction of polarization information, however, is challenging, both algorithmically and computationally. Specifically, the model functions used must be defined with care, and the issue of degrees of freedom in the map that are not determined by the differential DMR data must be addressed. Managing the size of the calculation is also important.

1. DMR and Polarization

The DMR instrument measures differences in signal between pairs of antennae 60° apart on the sky. The 53 and 90 GHz receivers are sensitive to linear polarization. The microwave signal measured by each antenna of a pair has its E-vector lying radially outward from the spacecraft spin axis.

Because the DMR instrument is differential and every pixel is multiply sampled, sky maps are produced by solving a coupled system of linear equations (the normal equations) to minimize a chi-squared. This procedure can be extended to account for the additional degrees of freedom represented by polarization. The signal observed by a radiometer through an antenna pointed at a particular pixel varies with the orientation angle α as $\mathrm{S}(\alpha) = \mathrm{I} + \mathrm{Q}\sin(2\alpha) + \mathrm{U}\cos(2\alpha)$. I, Q, and U are Stokes parameters representing the state of linear polarization, and α is the angle between a vector in the plane of the radiometers and a fiducial vector on the sky for this pixel. Since each pixel is observed with a range of values of α, recovery of polarization information is possible. We define our fiducial vector to point from the center of each pixel towards the north pole. (Our usage differs by a factor of two from Born & Wolf (1975) so that we can identify the sky map of I with the DMR sky map made ignoring polarization.)

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[4]The National Aeronautics and Space Administration/Goddard Space Flight Center (NASA/GSFC) is responsible for the design, development, and operation of the *Cosmic Background Explorer* (*COBE*). Scientific guidance is provided by the *COBE* Science Working Group. GSFC is also responsible for the development of the analysis software and for the production of the mission data sets.

2. Defining the Model Functions

From the formula for the signal detected at each pixel, one can construct and minimize a chi-squared to generate the normal equations whose solution gives maps of I, Q, and U on the sky. These normal equations are analogous to the equations for a conventional DMR unpolarized sky map, except that each pixel now has three parameters (I, Q, and U) rather than a single temperature. The correlation of these parameters means that there are nine times as many elements in the normal equations. For example, the single element representing the plus pixel (pixel viewed by the antenna which contributes positively to the difference) by minus pixel off-diagonal contribution to the normal equations for a single observation becomes the following 3 by 3 submatrix:

$$\begin{array}{ccc} -1 & -\sin(2\alpha_-) & -\cos(2\alpha_-) \\ -\sin(2\alpha_+) & -\sin(2\alpha_+)\sin(2\alpha_-) & -\sin(2\alpha_+)\cos(2\alpha_-) \\ -\cos(2\alpha_+) & -\cos(2\alpha_+)\sin(2\alpha_-) & -\cos(2\alpha_+)\cos(2\alpha_-) \end{array}$$

where α_+ is the angle for the plus antenna, and α_- the same for the minus antenna.

The normal equations matrix is the sum, for all observations, of these contributions. The condition required to obtain a useful solution is that each time the same pair of pixels are observed, the same values of α must be used in evaluating the contribution to the normal equations. Note that the orientation of the spacecraft, and hence the angles α for the plus and minus antennae, are fixed by specifying the pointing of the plus and the minus antennae. Thus one way to satisfy the condition for obtaining a useful solution is by specifying that the angles α be calculated on the assumption that the antennae are pointing towards the centers of the pixels they are piercing.

The effect of this condition on the normal equations is to make the off-diagonal elements of the final summed matrix integral multiples of the above 3 by 3 matrix, where the multiplier is the number of times that the two pixels were observed together. Note that all the columns and rows of the above explicit 3 by 3 matrix are multiples of each other. This condition would not hold in the final summed matrix if not all α_+ and α_- were equal.

3. Accounting for the Null Eigenvectors

Since the DMR instrument is differential, it must follow that it cannot measure the absolute level of the sky temperature. Phrased another way, the average temperature of the pixels in a conventional DMR unpolarized skymap is not constrained by the DMR data. This is reflected in the algebraic form of the normal equations matrix, so that, if one were to diagonalize the matrix, one would find a single eigenvector whose eigenvalue was identically zero, and the associated eigenvector, viewed as a sky map, would have a constant value.

The added symmetry of the polarization normal equations means that in this case there are six null eigenvectors, the above mentioned map with constant I, and five additional maps with more complicated patterns. These maps are shown in the figures; since eigenvectors are dimensionless patterns, the grayscale value represents arbitrary units.

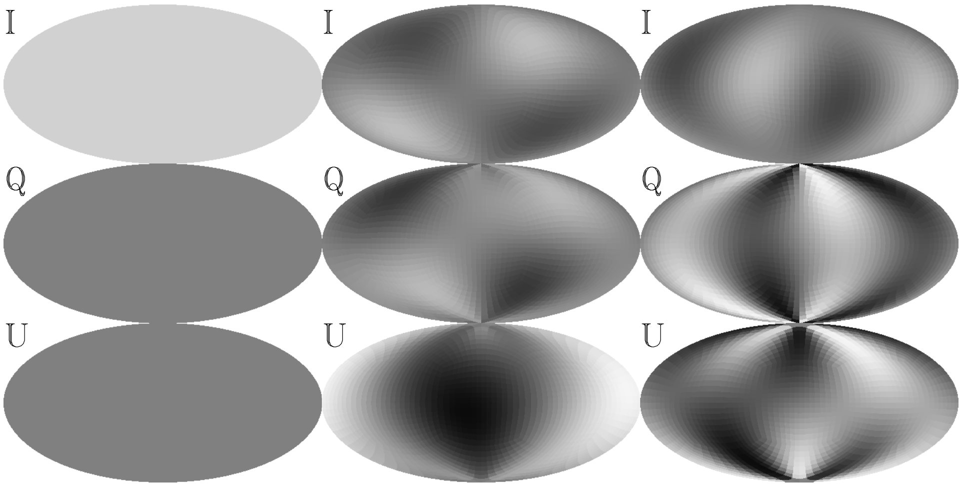

Figure 1. First, second, and third null eigenvectors. Each vector consists of projections of the full sky for each of I, Q, and U.

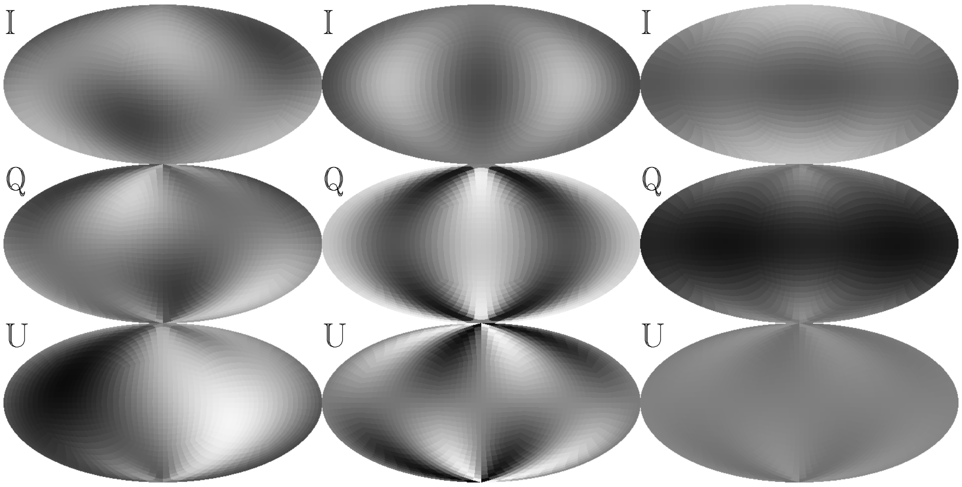

Figure 2. Fourth, fifth, and sixth null eigenvectors.

Care must be taken to preserve the symmetry which gives rise to these null eigenvectors, since this represents a genuine ambiguity in the experiment. Forcibly breaking the symmetry by using different values of α for different observations of the same pair of pixels will not produce any new information, but rather will cause large amounts of these null eigenvectors to couple into the final output map.

The condition imposed by the iterative solution algorithm is approximately that the projection of the null eigenvectors on the I, Q, and U maps taken together, is zero. This can leave large contributions from the null eigenvectors in the Q and U maps, so long as these are offset in the I map. Since we can calculate the I map much more reliably without polarization, what we are most interested in from these calculations is maps of Q and U. Consequently, as our final step in producing polarized maps, we explicitly project out the contribution of the null eigenvectors over the subspace consisting of just the Q and U maps.

4. Conclusions

The 53 and 90 GHz radiometers of the *COBE* DMR instrument are sensitive to linear polarization. It is straightforward to define the matrix system whose solution gives best fit solutions for I, Q, and U skymaps. However, the fact that these equations require a 3 by 3 floating-point submatrix rather than a single integer for each off-diagonal pair means that the memory requirements for solving these systems at the standard DMR resolution, using 6144 pixels, are not achievable. Since the memory required scales as the number of pixels to the three-halves power, reducing the size of the calculation one step, to 1536 pixels, allows the calculation to be tractable. To minimize the effect of temperature gradients over these larger pixels, and also to reduce cross-talk into the Q and U maps, the best estimate of the sky temperature from a standard resolution map is subtracted from each data point.

The correct prescription for formulating the model functions associated with each observation has been obtained. The null eigenvectors, which describe particular sky patterns which cannot be detected by DMR due to its differential nature, have been characterized, and a prescription for producing Q and U maps minimizing the contribution from these eigenvectors has been implemented. This projection operation is straightforward, and can be applied rather easily to Monte Carlo simulations of models for the polarized sky, thus facilitating comparison between models and the mission data.

References

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